Name: Solutions

Date: 3/17/2025

Math 130
Exam 2

Please show ALL your work on the problems below. No more than 1 point will be given to problems if you only provide the correct answer and insufficient work.

1. (20 points) Suppose you are going to make a bet with your friend on the result of drawing a single card from a standard poker deck. Specifically, you will win \$50 if you draw a red face card, you will lose \$10 if you draw any other red card, and you will lose \$5 is you draw any other card. Let *X* denote the amount of money you will win when playing this game once.

a) Find the probability distribution of *X*

x	$P(X=x)$
\$50	6/52
-\$10	20/52
-\$5	26/52

b) Find the expected value of *X*

$$\mu = \sum x P(X=x)$$
$$= (50)\left(\frac{6}{52}\right) + (-10)\left(\frac{20}{52}\right) + (-5)\left(\frac{26}{52}\right)$$
$$= \boxed{-\$0.58}$$

(unrounded $-0.5769230\ldots$)

c) Find the standard deviation of *X*

$$\sigma = \sqrt{\sum x^2 P(X=x) - \mu^2} = \sqrt{(50)^2\left(\frac{6}{52}\right) + (-10)^2\left(\frac{20}{52}\right) + (-5)^2\left(\frac{26}{52}\right) - (-0.5769\ldots)^2}$$

$$= \boxed{\$18.41}$$

d) Explain the meaning of your answer from part (b)

If you make this bet with a friend many times, it's as if you lose about \$0.58 per game.

2. (3, 3, 6, 9, 6 points) Stella the cook is good at burning the meals she prepares. In fact, the probability that she burns a meal is 28%. Assume that Stella burning a given meal is independent of the other times she burns a meal. Let X denote the number meals Stella burns among the next 21 meals she prepares.

a) What distribution does X have?

Binomial

b) Find the other 6 things you are supposed to list when solving problems for this kind of random variable.

$n = 21$

$p = 0.28$

Success = Stella burns a meal

$q = 0.72$

Failure = Stella doesn't burn a meal

X = The total number of meals Stella ~~[illegible]~~ burns among the next 21 meals she makes

c) What is the probability that Stella burns exactly 7 meals?

$$P(X=7) = {}_{21}C_7\,(0.28)^7(0.72)^{21-7} = \boxed{0.1578578533}$$

d) What is the probability that Stella burns between 7 and 9 meals (inclusive)?

$$P(X=7 \text{ or } X=8 \text{ or } X=9) = P(X=7) + P(X=8) + P(X=9)$$

$$= {}_{21}C_7\,(0.28)^7(0.72)^{21-7} + {}_{21}C_8\,(0.28)^8(0.72)^{21-8} + {}_{21}C_9\,(0.28)^9(0.72)^{21-9}$$

$$= 0.1579 + 0.1074 + 0.0603 = \boxed{0.3256}$$

e) What is the expected value, standard deviation, and variance of X?

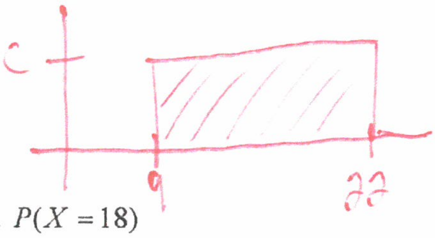
$$\mu = np = 21(0.28) = 5.88$$

$$\sigma^2 = npq = 21(0.28)(0.72) = 4.2336$$

$$\sigma = \sqrt{npq} = \sqrt{21(0.28)(0.72)} = 2.05757$$

3. (7, 5, 7 points) Suppose the random variable X has a uniform distribution on the interval [9, 22].

a) Find the value of c that makes this a probability distribution



Total area = 1

$b \cdot h = 1$

$13 \cdot c = 1$

$\frac{13c}{13} = \frac{1}{13}$

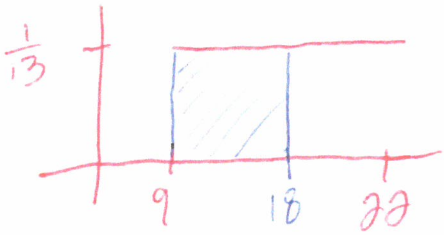
$\boxed{c = \frac{1}{13}}$

b) Find $P(X = 18)$

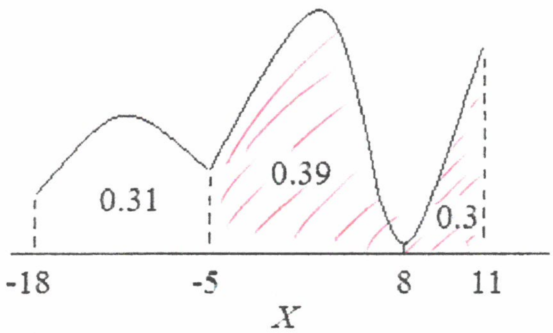
$= \boxed{0}$

c) Find $P(2 < X < 18)$

$= P(9 < x < 18) = b \cdot h = 9 \cdot \frac{1}{13} = \boxed{\frac{9}{13}}$



4. (14 points) Suppose X is a random variable whose density curve is given below.



a) What are the possible values of X?

All real numbers between -18 and 11

b) Find $P(-5 < X < 21)$

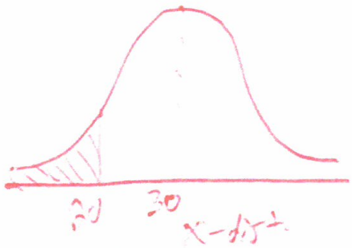
$= P(-5 < x < 11)$ = area shaded in picture above

$= 0.39 + 0.3 = \boxed{0.69}$

5. (28 points) The time it takes me to grade a stack of stats quizzes has a normal distribution with a mean of 30 minutes and a standard deviation of 6 minutes.

a) What is the probability that the next time I grade a stack of stats quizzes it will take me at most 20 minutes?

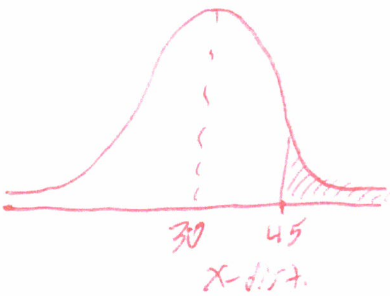
$$P(X<20) \overset{z\text{-trans.}}{=} P\left(\frac{X-\mu}{\sigma} < \frac{20-\mu}{\sigma}\right) = P\left(Z < \frac{20-30}{6}\right)$$



$$= P(Z<-1.67) = \boxed{0.0475}$$

b) What is the probability that the next time I grade a stack of stats quizzes it will take me more than 45 minutes?

$$P(X>45) \overset{z\text{-trans}}{=} P\left(\frac{X-\mu}{\sigma} > \frac{45-\mu}{\sigma}\right) = P\left(Z > \frac{45-30}{6}\right)$$

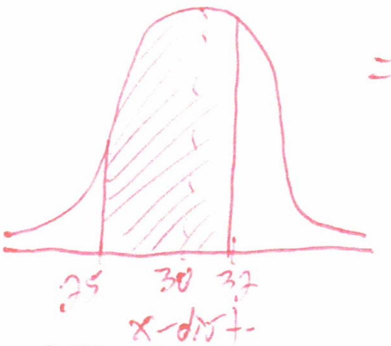


$$= P(Z>2.50) = 1-P(Z<2.50)$$

$$= 1-0.9938 = \boxed{0.0062}$$

c) What is the probability that the next time I grade a stack of stats quizzes it will take me between 25 minutes and 32 minutes?

$$P(25<X<32) \overset{z\text{-trans}}{=} P\left(\frac{25-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{32-\mu}{\sigma}\right)$$



$$= P\left(\frac{25-30}{6} < Z < \frac{32-30}{6}\right) = P(-0.83 < Z < 0.33)$$

$$= P(Z<0.33) - P(Z<-0.83) = 0.6293 - 0.2033$$

$$= \boxed{0.426}$$

d) What does the probability you found in part (b) mean?

If I grade a stack of quizzes many times, it will take me more than 45 minutes to finish grading them about 0.62% of the time.

6. (5, 7 points) Consider the experiment where in order to complete the experiment once you have to first flip a single coin then roll a single die.

a) What is the sample space?

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

b) Define a random variable on this experiment.

$X = \begin{cases} 1 & \text{if coin lands on } H \\ 2 & \text{if coin lands on } T \end{cases}$

or

$Y =$ The # on the die (ignoring the coin flip)

or

$Z = \begin{cases} 1 & \text{for } H1 \text{ and } T6 \\ 2 & \text{for } H3, H5, \text{ and } T4 \\ 7 & \text{otherwise.} \end{cases}$

Some formulas you may need:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad P(A \cup B) = P(A) + P(B)$$

$$P(A \cap B) = P(A) \cdot P(B \mid A) \qquad P(A \cap B) = P(A) \cdot P(B)$$

$$P(\overline{A}) = 1 - P(A) \qquad P(at \quad least \quad one) = 1 - P(none)$$

$$EV = \mu = \sum xp(X = x)$$

$$Var = \left[\sum x^2 p(X = x)\right] - \mu^2$$

$$\sigma = \sqrt{\left[\sum x^2 p(X = x)\right] - \mu^2}$$

$$P(X = x) = {}_nC_x p^x q^{n-x} \qquad \mu = np \qquad \sigma^2 = npq \qquad \sigma = \sqrt{npq}$$

$$Z = \frac{X - \mu}{\sigma}$$